University of Kurdistan
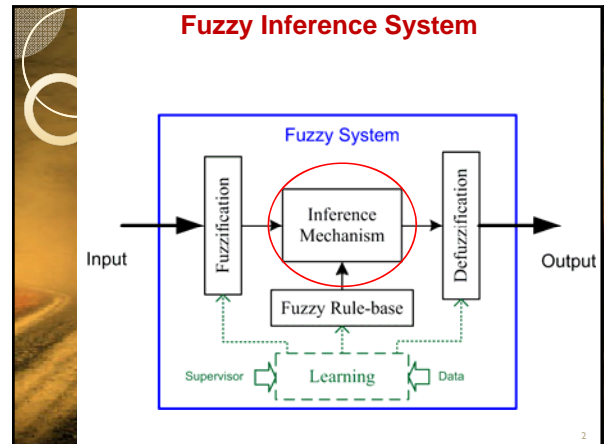**Department of Electrical & Computer Eng.**

Fuzzy Inference Systems Mamdani & Sugeno

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Fuzzy Inference System

Inference systems

Important Inference systems:

Mamdani
Sugeno

Mamdani Fuzzy Inference

- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.

- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.

An Example

We examine a simple two-input one-output problem that includes three rules:

Rule: 1
IF   x is A3
OR   y is B1
THEN z is C1

Rule: 1
IF   project_funding   is adequate
OR   project_staffing   is small
THEN risk   is low

Rule: 2
IF   x is A2
AND   y is B2
THEN z is C2

Rule: 2
IF   project_funding   is marginal
AND   project_staffing   is large
THEN risk   is normal

Rule: 3
IF   x is A1
THEN z is C3

Rule: 3
IF   project_funding   is inadequate
THEN risk   is high

Step 1: Fuzzification

- The first step is to take the crisp inputs, x1 and y1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.

Step 2: Rule Evaluation

- The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules.

- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

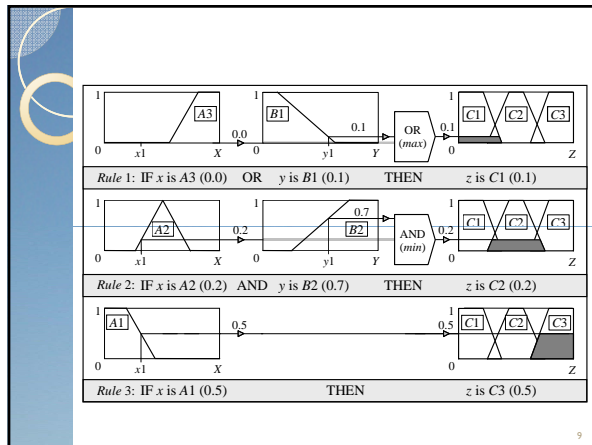- This number (the truth value) is then applied to the consequent membership function.

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To evaluate the disjunction of the rule antecedents, we use the OR fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

$$\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the AND fuzzy operation intersection:

$$\mu_{A \cap B}(x) = \min [\mu_A(x), \mu_B(x)]$$

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Rule 1: IF x is A3 (0.0)   OR   y is B1 (0.1)   THEN   z is C1 (0.1)

Rule 2: IF x is A2 (0.2)   AND   y is B2 (0.7)   THEN   z is C2 (0.2)

Rule 3: IF x is A1 (0.5)   THEN   z is C3 (0.5)

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- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.

- There are two main methods for doing so:
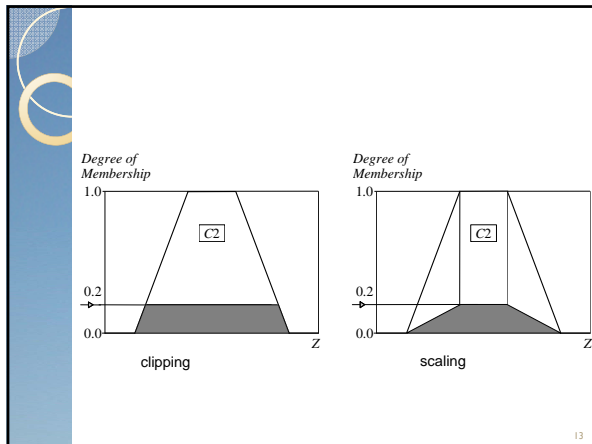 - Clipping
  - Scaling

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- The most common method of correlating the rule consequent with the truth value of the rule antecedent is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).

- Since the top of the membership function is sliced, the clipped fuzzy set loses some information.

- However, clipping is still often preferred because it involves less complex and faster mathematics, and generates an aggregated output surface that is easier to defuzzify.

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- While clipping is a frequently used method, **scaling** offers a better approach for preserving the original shape of the fuzzy set.
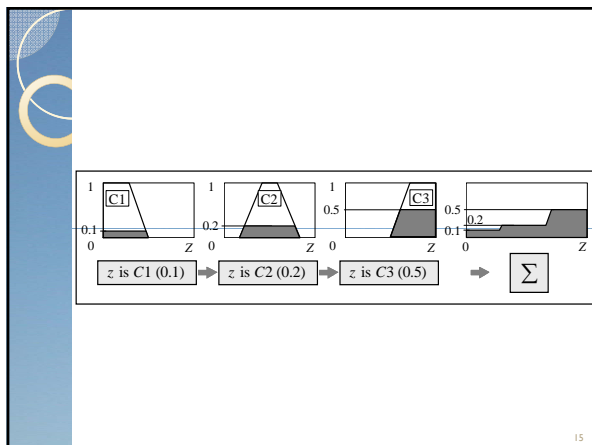
- The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.

- This method, which generally loses less information, can be very useful in fuzzy expert systems.

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Degree of Membership
1.0
0.2
0.0

C2

clipping

z

Degree of Membership
1.0
0.2
0.0

C2

scaling

z

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Step 3: Aggregation of the rule outputs

- Aggregation is the process of unification of the outputs of all rules.

- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.

- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.

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1
0.1
0

C1

z

1
0.2
0

C2

z

1
0.5
0

C3

z

0.5
0.1

0.2

z

z is $C1$ (0.1) → z is $C2$ (0.2) → z is $C3$ (0.5) → Σ

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Sugeno Fuzzy Inference

- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.

- Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.

- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

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- Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable. The format of the **Sugeno-style fuzzy rule** is
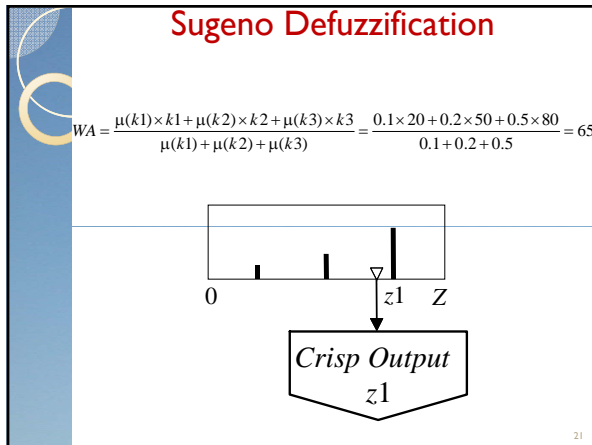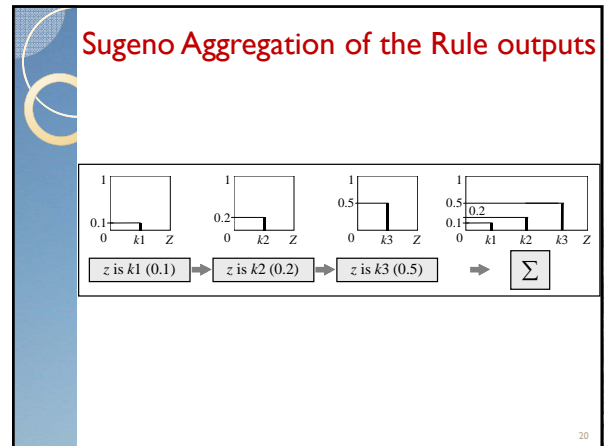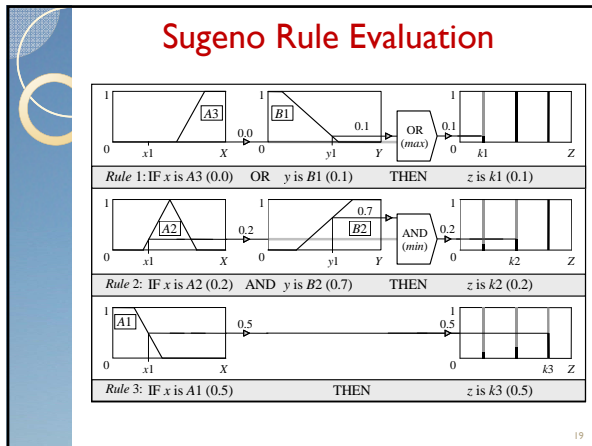
  IF x is A
  AND y is B
  THEN z is $f(x, y)$

  where x, y and z are linguistic variables; A and B are fuzzy sets on universe of discourses X and Y, respectively; and $f(x, y)$ is a mathematical function.

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- The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

  IF x is A
  AND y is B
  THEN z is k

  where k is a constant.

- In this case, the output of each fuzzy rule is constant. All consequent membership functions are represented by singleton spikes.

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Sugeno Rule Evaluation

Rule 1: IF x is A3 (0.0)   OR   y is B1 (0.1)   THEN   z is k1 (0.1)

Rule 2: IF x is A2 (0.2)   AND   y is B2 (0.7)   THEN   z is k2 (0.2)

Rule 3: IF x is A1 (0.5)   THEN   z is k3 (0.5)

Sugeno Aggregation of the Rule outputs

z is k1 (0.1)   z is k2 (0.2)   z is k3 (0.5)   →   Σ

Sugeno Defuzzification

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

Crisp Output z1

Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.

- On the other hand, Sugeno method is computationally effective and works well with optimisation and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.