Slide 0

Chap.6 Basic Formalisms: DEVS, DTSS, DESS

Slide 1

Discrete event system specification (DEVS)

- formalisms
  - a means of specifying a system
  - a local description of the dynamic behavior of the system →global dynamic behavior of the system
- DEVS

  $DEVS = (X, Y, S, \delta_{int}, \delta_{ext}, \lambda, ta)$

  where

  X is the set of inputs
  Y is the set of outputs
  S is a set of sequential states
  $\delta_{int} : S \to S$ is the *internal state transition* function
  $\delta_{ext} : Q \times X \to S$ is the *external state transition* function, where
  $Q = \{(s, e) | s \in S, 0 \leq e \leq ta(s)\}$ is the set of *total states*
  e is the *time elapsed* since last transition
  $\lambda : S \to Y$ is the output function
  $ta : S \to \Re^{+}_{0,\infty}$ is the time advance function

Discrete event system specification (DEVS) (Cont'd)

Slide 2

- structure S specified by a DEVS is time invariant
  - its time base T is real numbers $\mathfrak{R}$ (subset such as integers are also allowed)
  - its input set is $X^{\varnothing} = X \cup \{\varnothing\}$
  - its output set is $Y^{\varnothing} = Y \cup \{\varnothing\}$
  - its state set is the total state set Q of the DEVS
  - the set of Ω of admissible input segments is the set of all DEVS segments over X and T
  - the state trajectories are piecewise constant segments over S and T
  - the output trajectories are DEVS segments over Y and T
  - the state trajectory is defined as follows
    - let $\omega :< t_i, t_f] \rightarrow X^{\varnothing}$ be a DEVS segment and the state $q = (s, e)$ be the state at time t_i, then we define
      $\Delta(q, \omega < t_i, t_f]) =$
      (1) $(s, e + t_f - t_i)$ if $e + t_f - t_i < ta(s) \wedge \neg\exists t \in < t_i, t_f] : \omega(t) \neq \varnothing$ (no internal and external events)
      (2) $\Delta((\delta_{int}(s), 0), \omega[t_i + ta(s) - e, t_f])$ if $e + t_f - t_i = ta(s) \wedge \neg\exists t \in < t_i, t_i + ta(s) - e) : \omega(t) \neq \varnothing$ (an internal event)

Discrete event system specification (DEVS) (Cont'd)

Slide 3

(3) $\Delta((\delta_{ext}((s, e + t - t_i), \omega(t)), 0), \varnothing[t, t] \bullet \omega[t, t_f])$ if $\exists t \in < t_i, \min\{t_f, t_i + ta(s) - e\}] : \omega(t) \neq \varnothing \wedge \neg\exists t' \in < t_i, t) : \omega(t') \neq \varnothing$ (an external event)

(4) the output function Λ is given by

$$\begin{aligned} \Lambda(q, x) &= \lambda(s) && \text{if } e = ta(s) \text{ and } \omega(t) = \varnothing \\ &= \lambda(s) \text{ or } \varnothing && \text{if } e = ta(s) \text{ and } \omega(t) \neq \varnothing \\ &= \varnothing && \text{otherwise.} \end{aligned}$$

- an internal event and an external event at the same time
  - dynamic behavior is not uniquely defined when both an internal event and an external event occur at the same time
  - two approaches
    - in classic DEVS, select function allows only one component to be activated $\rightarrow$no collision
    - in parallel DEVS, additional transition specification that deals with the collision directly

Discrete event system specification (DEVS) (Cont'd)

Slide 4

- legitimacy: when is the structure specified by a DEVS really a system?
  - iterative form of internal transition function

    $\delta^+_{int} : S \times I^+_0 \rightarrow S$

    $\delta^+_{int}(s, 0) = s$, $\delta^+_{int}(s, n+1) = \delta_{int}(\delta^+_{int}(s, n))$
  - the time accumulate function $\sum(s, n)$

    $\sum : S \times I^+_0 \rightarrow R^+_0$

    $\sum(s, 0) = 0$, $\sum(s, 0) = \sum_{i=0}^{n-1} ta(\delta^+_{int}(s, i))$
  - definition: a DEVS is legitimate if for each $s \in S, lim_{n \rightarrow \infty} \sum(s, n) \rightarrow \infty$
  - theorem 1
    - the structure specified by a DEVS is a system if, and only if, the DEVS is legitimate
  - theorem 2
    - a DEVS M is legitimate under the following conditions

      (a) M is finite (S is a finite set): every cycle in the state diagram of δ_{int} contains a nontransitory state

      (b) M is infinite: there is a positive lower bound on the time advances, i.e., $\exists b \forall s \in S, ta(s) > b$

Discrete event system specification (DEVS) (Cont'd)

Slide 5

- parallel DEVS

  $DEVS = (X, Y, S, \delta_{ext}, \delta_{int}, \delta_{con}, \lambda, ta)$

  where

  X is the set of input events

  Y is the set of output events

  S is the set of sequential states

  $\delta_{int} : S \rightarrow S$ is the *internal transition* function

  $\delta_{ext} : Q \times X^b \rightarrow S$ is the *external transition* function, where X^b is a set of bags

  $\delta_{con} : Q \times X^b \rightarrow S$ is the *confluent transition* function, subject to $\delta_{con}(s, \varnothing) = \delta_{int}(s)$

  $\lambda : S \rightarrow Y^b$ is the output function

  $ta : S \rightarrow \mathfrak{R}^+_{0,\infty}$ is the *time advance* function

  $Q = \{(s, e) | s \in S, 0 \leq e \leq ta(s)\}$ is the set of *total states* and e is the elapsed time since last state transition

Discrete event system specification (DEVS) (Cont'd)

Slide 6

- extensions
  - allow bags of inputs to the external transition function (a bag is similar to a set except that multiple occurrences are allowed, e.g., $\{a, b, a, c\}$ is a bag)
  - confluent transition function, δ_{con}
    - rather than serializing through select function, leaves this decision to individual component

  (1) $\delta_{con}(s, x) = \delta_{ext}(\delta_{int}(s), 0, x)$

  (2) $\delta_{con}(s, x) = \delta_{int}(\delta_{ext}(s, ta(s), x))$

  (3) the others for special circumstances

Discrete time system specification (DTSS)

Slide 7

- DTSS

  $DTSS = (X, Y, Q, \delta, \lambda, c)$

  where

  X is the set of inputs
  Y is the set of outputs
  Q is the set of states
  $\delta : Q \times X \rightarrow Q$ is the *state transition* function
  $\lambda : Q \rightarrow Y$ is the Moore-type output function
  $\lambda : Q \times X \rightarrow Y$ is the Mealy-type output function
  c is a constant employed for the specification of the time base $c \bullet \mathfrak{I}$

  - structure specified by the DTSS is a structure
  - $S = (T, X, \Omega, Y, Q, \Delta, \Lambda)$, where
    - the time base T has to be the set $c \bullet \mathfrak{I}$ isomorphic to the integers
    - X, Y, and Q can be arbitrary sets
    - the set Ω of admissible input segments is the set of all sequences over X and T

Discrete time system specification (DTSS) (Cont'd)

Slide 8

* the state and output trajectories are sequences over Q and T and Y and T, respectively
* a DTSS defines the following dynamic behavior
  - given an input segment $\omega : [t_1, t_2) \rightarrow X$ and an initial state q at time t_i, then we define the global transition function Δ by

$$\begin{aligned} \Delta(q, w) &= q & \text{if } t_1 = t_2 \\ &= \delta(q, \omega(t_1)) & \text{if } t_1 = t_2 - c \\ &= \delta(\Delta(q, \omega_{t_2}^{-c}), \omega(t_2 - c)) & \text{otherwise} \end{aligned}$$

  - the output function Λ is given by $\Lambda(q, x) = \lambda(q)$ for Moore-type and $\Lambda(q, x) = \lambda(q, x)$ for Mealy-type

Differential equation system specification (DESS)

Slide 9

- DESS

  $DESS = (X, Y, Q, f, \lambda)$

  where

  X is the set of inputs
  Y is the set of outputs
  Q is the set of states
  $f : Q \times X \rightarrow Q$ is the *rate of change* function
  $\lambda : Q \rightarrow Y$ is the Moore-type output function
  $\lambda : Q \times X \rightarrow Y$ is the Mealy-type output function

  – structure specified by the DESS is a structure
  – $S = (T, X, \Omega, Y, Q, \Delta, \Lambda)$, where
    * the time base T has to be the real number $\mathfrak{R}$
    * X, Y, and Q are real-valued vector spaces $\mathfrak{R}^m, \mathfrak{R}^p, \mathfrak{R}^n$, respectively
    * the set Ω of admissible input segments is the set of all bounded piecewise continuous input segments

Differential equation system specification (DESS) (Cont'd)

Slide 10

* the state and output trajectories are bounded piecewise continuous trajectories
* a DESS defines the following dynamic behavior
  - given a bounded continuous input segment $\omega :< t_1, t_2 > \rightarrow X$ and an initial state q at time t_i, then

$$\begin{aligned} STRAJ_{q,\omega}(t_1) &= q \\ dSTRAJ_{q,\omega}(t)/dt &= f(STRAJ_{q,\omega}(t), \omega(t)) \end{aligned}$$

  transition function: $\Delta(q, \omega) = STRAJ_{q,\omega}(t_2)$
  - the output function Λ is given by $\Lambda(q, x) = \lambda(q)$ for Moore-type and $\Lambda(q, x) = \lambda(q, x)$ for Mealy-type